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# Abelian duality, confinement, and chiral symmetry breaking in QCD(adj)

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We analyze the vacuum structure of $SU(2)$ QCD with multiple massless adjoint representation fermions formulated on a small spatial $S^1 \times \mathbb{R}^3$. The absence of thermal fluctuations, and the fact that quantum fluctuations favoring the vacuum with unbroken center symmetry in a weakly coupled regime renders the interesting dynamics of these theories analytically calculable. Confinement, the area law behavior for large Wilson loops, and the generation of the mass gap in the gluonic sector are shown analytically. By abelian duality transformation, the long distance effective theory of QCD is mapped into an amalgamation of $d = 3$ dimensional Sine-Gordon and NJL models. The duality necessitates going to IR first. In this regime, theory exhibits confinement without continuous chiral symmetry breaking. However, a flavor singlet chiral condensate (which breaks a discrete chiral symmetry) persists at arbitrarily small $S^1$. Under the reasonable assumption that the theory on $\mathbb{R}^4$ exhibits chiral symmetry breaking, there must exist a zero temperature chiral phase transition in the absence of any change in spatial center symmetry realizations.

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This letter aims to address confinement, mass gap in gluonic sector, and chiral symmetry realizations in certain locally four dimensional, asymptotically free QCD-like gauge theories. To date, confinement is understood quantitatively in a subclass of non-abelian gauge theories, such as Polyakov's model on $\mathbb{R}^3$ [1], and in mass deformation of $\mathcal{N} = 2$ SYM on $\mathbb{R}^4$ [2]. The common feature of both theories is the presence of elementary scalars in the defining Lagrangian and gauge symmetry breaking. In both cases, confinement occurs via monopole condensation. The QCD-like theories in four dimensions lack elementary scalars, and therefore are conceptually harder. (See Refs.[3, 4, 5] for reviews.) We hope to provide useful insights into such theories on $\mathbb{R}^3 \times S^1$, ($\mathbb{R}^{2,1} \times S^1$ in the Minkowski setting).

In QCD-like theories formulated on small $S^1 \times \mathbb{R}^3$ (hence weak coupling), the Wilson line along the compact direction may be viewed as an adjoint Higgs field. In cases where $S^1$ is thermal, the thermal fluctuation causes the center symmetry to break and the theory is in deconfined phase [6]. It would be nice if the center symmetry was not broken at small $S^1$. However, at high temperature, this is not debatable, since thermal fluctuations will necessarily overwhelm the center symmetry. We want the benefit of weak coupling (small $S^1$), and the lack of thermal fluctuations. Therefore, we consider zero temperature QCD at small spatial $S^1 \times \mathbb{R}^3$. Since we do not a priori know what the effect of zero temperature quantum fluctuations will be, we should be ready for surprises, and novel phenomena.

Remarkably, QCD with $n_f$ adjoint Weyl fermions [abbreviated as QCD(adj)] with periodic spin connection along the $S^1$ respects its center symmetry in a weakly coupled regime [7]. The benefit of weak coupling is that gauge symmetry is broken, and unlike the thermal case, the long distance theory abelianizes. By quantizing $SU(2)$ QCD(adj) on small $S^1 \times \mathbb{R}^3$, we exhibit

i) Permanent confinement, the area law behavior for Wilson loops,

ii) Absence of continuous chiral symmetry breaking,

iii) Presence of a flavor singlet chiral condensate which only breaks the discrete chiral symmetry,

iv) The existence of a mass gap in the gluonic sector, and massless fermions in the spectrum

With the assumption that the theory on large $S^1 \times \mathbb{R}^3$ exhibits chiral symmetry breaking, ii) implies

v) The existence of a chiral phase transition in the absence of any change in center symmetry

This is a zero temperature phase transition triggered solely by quantum fluctuations.

**Perturbation theory and spatial center symmetry:** The action of $SU(2)$ QCD(adj) defined on $\mathbb{R}^3 \times S^1$ is

$$S = \int_{\mathbb{R}^3 \times S^1} \frac{1}{g^2} \mathrm{tr} \left[ \frac{1}{4} F_{MN}^2 + i \bar{\lambda}^I \bar{\sigma}^M D_M \lambda_I \right] \qquad (1)$$

where $\lambda_I = \lambda_{I,a} t_a, a = 1, 2, 3$ is Weyl fermion in adjoint representation, $F_{MN}$ is the nonabelian gauge field strength, and $I$ is the flavor index. Classically, the theory possess an $U(n_f)$ flavor symmetry whose $U(1)_A$ part is anomalous. The symmetry of the quantum theory is $SU(n_f) \times \mathbb{Z}_{4n_f}$. The quantum theory has the dynamical strong scale $\Lambda$, which arises via dimensional transmutation, and is given by $\Lambda^{b_0} = \mu^{b_0} e^{-4\pi^2/g^2(\mu)}$ where $\mu$ is the renormalization group scale and $b_0 = (11 - 2n_f)/3$. We consider $1 < n_f \leq 4$ so that asymptotic freedom is preserved and the theories are nonsupersymmetric.

At small $S^1$ ($L\Lambda \ll 1$), due to asymptotic freedom, the gauge coupling is small and a perturbative Coleman-Weinberg analysis is reliable [8]. The minimum of the gauge field action corresponds to the vanishing field

strength, and constant but arbitrary values of the Wilson line

$$U = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} . \tag{2}$$

Integrating out the heavy KK-modes along the $S^1$ circle, $|\omega_n| \geq \omega_1$ where $\omega_n = \frac{2\pi}{L}n, n \in \mathbb{Z}$, induce a nontrivial effective potential for $U$, given by (up to an uninteresting constant)

$$V_{\text{eff}}[U] = -\frac{(n_f - 1)}{24\pi^2 L^4}[2\phi]^2([2\phi] - 2\pi)^2 \tag{3}$$

where $\phi \equiv \phi + 2\pi$ is a periodic variable. The potential is bounded. The action for the classical zero modes reduce to

$$S = \int_{\mathbb{R}^3} \frac{L}{g^2}\text{tr} \; \Big[\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}(D_\mu \Phi)^2 + g^2 V(|\Phi|) \\ + i\bar{\lambda}^I(\bar{\sigma}^\mu D_\mu + \bar{\sigma}_4[\Phi, \;])\lambda_I\Big] \tag{4}$$

The minimum of the potential $V_{\text{eff}}$ is located at $|\Phi| \equiv \phi = \frac{\pi}{2}$, hence $U = \text{Diag}(e^{i\pi/2}, e^{-i\pi/2})$. Since $\text{tr}U = 0$, the $\mathbb{Z}_2$ center symmetry is preserved. By Higgs mechanism, the gauge symmetry is broken down as

$$SU(2) \to U(1) \tag{5}$$

*Remark:* This is unlike thermal QCD(adj), in which the minimum of the potential for the thermal Wilson line is located at $U = \pm 1$ and center symmetry is spontaneously broken. The gauge symmetry remains unbroken, and the theory reduces to non-abelian $d = 3$ dimensional pure $SU(2)$ Yang-Mills theory at long distances.

Due to the gauge symmetry breaking via an "adjoint Higgs field", the neutral fields aligned with $U$ along the Cartan subalgebra $(A_{3,\mu}, \lambda_3^I)$ remain massless, and off-diagonal components acquire mass, given by the separation between the eigenvalues of the Wilson line

$$m_{W^\pm} = m_{\lambda^{I,\pm}} = (\phi_1 - \phi_2)/L = \pi/L \tag{6}$$

where $\pm$ refers to the charges under unbroken $U(1)$. Since higher order perturbative effects cannot change the conclusion about the center symmetry realizations, within perturbation theory, the low energy theory is a $d = 3$ dimensional abelian $U(1)$ gauge theory with $n_f$ massless fermions with a free action

$$S = \int_{\mathbb{R}^3} \frac{L}{g^2}\left[\frac{1}{4}F_{3,\mu\nu}^2 + i\bar{\lambda}_3^I \bar{\sigma}^\mu \partial_\mu \lambda_{3,I}\right] \tag{7}$$

At distances shorter than $L$, the coupling constant flows according to the four dimensional renormalization group. Since the heavy $W^\pm, \lambda^{I,\pm}$ which are charged under $U(1)$ decouple from the long distance physics at scale $L$ and above, the coupling constant ceases to run at $1/L \gg \Lambda$ much before the strong coupling sets in. In perturbation theory, this is the whole story.

Nonperturbatively though, the free infrared fixed point is unstable. This follows from the existence of monopoles. The distinction between the free $U(1)$ theory, and the theory with monopoles is that in the latter the $U(1)$ symmetry enhances to the whole $SU(2)$ at the monopole cores. Below, we will demonstrate that, due to nonperturbative effects, QCD(adj) exhibits confinement. However, we first have to take a detour and answer the following question:

**Is this Polyakovs model with adjoint fermions on $\mathbb{R}^3$?** The action Eq.4 looks "almost" like the generalization of the Polyakov model on $\mathbb{R}^3$ in the presence of $n_f$ Dirac fermions in adjoint representation, with one difference: the compact adjoint Higgs field in Eq.4 has to be substituted by a non-compact one.

$$V_{\text{eff}}^{\text{compact}}(|\Phi|) \to V_{\text{eff}}^{\text{noncompact}}(|\Phi|) \tag{8}$$

Such extensions are studied in Ref. [9] and do **not** exhibit confinement. What is going on? What is the conceptual difference between the two which results in such drastically different physics?

The simplest explanation is through the symmetries. Let me give the microscopic explanation. In odd dimensions, there is no chiral anomaly. Hence, the theory on $\mathbb{R}^3$ has a genuine $U(n_f)$ flavor symmetry whose $U(1)$ part is just fermion number. Since the QCD(adj) theory on small $S^1 \times \mathbb{R}^3$ is locally four dimensional, it has only $SU(n_f) \times \mathbb{Z}_{4n_f}$ symmetry, where $\mathbb{Z}_{4n_f}$ is the anomaly free part the anomalous $U(1)_A$ symmetry. All the effective long distance theories must obey the symmetries of their microscopic origin. Hence, in particular, the $U(1)$ symmetry will be a symmetry of the effective theory corresponding to the extension of Polyakov's model, and $\mathbb{Z}_{4n_f}$ will be the one of QCD(adj).

In both theories, nonperturbatively, there exists topologically stable, semiclassical field configurations, i.e, monopoles. Their existence follows from the gauge symmetry breaking. Since the second homotopy group $\pi_2[SU(2)/U(1)] = \pi_1[U(1)] = \mathbb{Z}$, there is one type of almost BPS monopole. In contradistinction, in QCD(adj), there is also a KK-monopole which may be interpreted either due to compactness (or equivalently, due to the fact that the underlying theory is defined on $S^1 \times \mathbb{R}^3$.) Neither monopole is exactly BPS, and their action receives small $O(g^2)$ correction $S_i = \frac{4\pi^2}{g^2}(1 + O(g^2))$ which we will neglect. One important point is that the magnetic charge of the KK-monopole is opposite to that of the BPS monopole. There are also antimonopoles.

Now, let us review the construction of the long distance effective theory for the extension of Polyakov's model, see Ref. [9] for a full discussion. Let $\sigma$ denote the scalar dual to the photon obtained via the abelian duality. (The infrared physics is easier to describe in the dual description.) In the background of the BPS monopole, due to Callias index theorem there exists $2n_f$ fermionic zero modes [10, 11]. Hence, a manifestly $SU(n_f)$ invariant monopole induced fermion vertex should involve $\det \psi^I \psi^J$. This vertex is noninvariant under the $U(1)$

fermion number, and this can be cured by coupling to the dual photon. Generalizing the result of Ref.[9] to multiflavor ($n_f > 1$), we obtain the long distance effective Lagrangian as

$$L^{\rm n.c.}_{\rm eff} = \frac{1}{2}(\partial\sigma)^2 + i\bar{\psi}^I\gamma^\mu\partial_\mu\psi_I + ae^{-S_0}(e^{i\sigma}\det\psi^I\psi^J + {\rm c.c.}) \tag{9}$$

This is respectful to all the symmetries of the underlying theory: Most importantly, the fermion number symmetry

$$\psi^I \to e^{i\alpha}\psi^I, \;\; \bar{\psi}^I \to e^{-i\alpha}\bar{\psi}^I, \;\; \sigma \to \sigma - 2n_f\alpha \; . \tag{10}$$

The $U(1)$ symmetry prohibits any kind of mass term (or potential) for the dual photon. As shown in Ref.[9], the $U(1)$ symmetry is spontaneously broken down to $\mathbb{Z}_{2n_f}$ due to the monopole induced $\det\psi^I\psi^J$ term. (This can be seen by expanding the photon around $\sigma = 0$, for example.) Hence, there must be a Golstone boson associated with it. In three dimensions, there is no distinction between scalars and vectors, and the photon is the Goldstone boson of the spontaneously broken fermion number symmetry. Since the photon does not acquire a mass, it remains infinite range, i.e., there is no confinement.

Notice that in the effective lagrangians, we will always use dimensionless coordinates, fields ($\sigma$ and $\psi$) all measured in units of $L$. We will also not calculate various one loop factors in these lagrangians, such as $a, b, c$ which are calculable, but inessential for our conclusions.

**Abelian duality and dual QCD:** In QCD(adj), the $U(1)_A$ which may potentially prevent the photon from acquiring mass is not a real symmetry. Its anomaly free incarnation is $\mathbb{Z}_{4n_f}$ which can not prevent a mass term for the dual photon. Let us see this in detail. Just like the BPS monopole, the KK-monopole will also induce a determinantal fermion vertex, $e^{-i\sigma}\det\psi^I\psi^J$ where the relative minus sign reflects the fact that KK and BPS monopoles carry opposite charges. The combined effect of BPS and KK monopoles is $\cos(\sigma)\det\psi^I\psi^J$. This vertex is manifestly invariant under $SU(n_f)$, and respects the discrete symmetry

$$\psi^I \to e^{i2\pi/(4n_f)}\psi^I, \qquad \sigma \to \sigma + \pi \tag{11}$$

The presence of KK monopoles makes it impossible for the interaction vertex to be invariant under a continuous $U(1)$. This is how the $d = 3$ dimensional action "sees" that it has a hidden forth dimension, and it is genuinely different from a locally three dimensional theory. The simplest potential term for $\sigma$ allowed by the $\mathbb{Z}_2$ shift symmetry is $[e^{-S_0}\cos\sigma]^2 \sim e^{-2S_0}\cos 2\sigma$. Hence, the long distance effective theory which describes the dynamics of QCD(adj) on small $S^1 \times \mathbb{R}^3$ is

$$\begin{aligned} & L^{\rm dQCD} = \frac{1}{2}(\partial\sigma)^2 - b\, e^{-2S_0}\cos 2\sigma \\ + \quad & i\bar{\psi}^I\gamma_\mu\partial_\mu\psi_I + c\, e^{-S_0}\cos\sigma(\det_{I,J}\psi^I\psi^J + {\rm c.c.}) \end{aligned} \tag{12}$$

which is manifestly invariant under $SU(n_f) \times \mathbb{Z}_{4n_f}$ symmetry of the original theory.

**Mass gap in the gauge sector:** The small fluctuations around one of the minima of the $\cos 2\sigma$ potential shows that the dual photon acquires a mass, proportional to $e^{-S_0}$. This is the Debye mass in the classical plasma. In terms of the gauge theory, it is the inverse characteristic size of the chromoelectric flux tube. For $n_f$ flavor theory, it is given by

$$m_D \sim \Lambda(\Lambda L)^{b_0 - 1} = \Lambda(\Lambda L)^{(8-2n_f)/3} \; . \tag{13}$$

This is a remarkable result. It exhibits that the gauge sector of the QCD(adj) theory is quantum mechanically gapped due to non-perturbative effects. Since the chromoelectric fields become short range, this also implies confinement.

**Area law of confinement and monodromy:** We wish to exhibit the area law of confinement by calculating Wilson loops in the half-spin representations. The representation of the Wilson loops under the center group $\mathbb{Z}_2$ are in one to one correspondence with the monodromies, $\int_C d\sigma$ in the dual theory [12]. (Both are representation of $\mathbb{Z}_2$.) Evaluating the Wilson loops in a representation with odd or even $\mathbb{Z}_2$ center group charge assignment in the original theory translates into finding the field configurations for the dual scalar with monodromies equal to $\pi$ or 0 (mod $2\pi$), respectively. Therefore, we need to classify the vacuum states in the dual theory, and the soliton configurations interpolating between them. The Sine-Gordon potential has two gauge inequivalent vacua

$$|\Omega_0\rangle \equiv |\Omega_{0+2\pi k}\rangle, \qquad |\Omega_1\rangle \equiv |\Omega_{\pi+2\pi k}\rangle, \;\; k \in \mathbb{Z} \tag{14}$$

Therefore, the expectation values of the Wilson loop falls into two categories, for half-integer and integer spin representation. Let $H$ denote the Hamiltonian of the dual theory: The Hilbert space interpretation of Polyakov's result is

$$\lim_{A(\Sigma)\to\infty}\langle W_{\rm odd(even)}(C)\rangle|_{C=\partial\Sigma} = \langle\Omega_{1(0)}|e^{-zH}|\Omega_0\rangle \tag{15}$$

where $z$ is Euclidean time, and interpolates between $[-\infty, \infty]$. The expectation value of arbitrarily large Wilson loops (where $\Sigma$ is $\mathbb{R}^2$ filling) are equal to tunneling amplitudes in the dual theory. Formally, the tunneling amplitude on $\mathbb{R}^2 \times \mathbb{R}$ is $e^{-{\rm Area}(\mathbb{R}^2)S^*}$ where $S^*$ is the least action associated with $x, y \in \mathbb{R}^2$ independent soliton (kink) solution. The kink is localized within the $m_D^{-1}$ proximity of the surface $\Sigma$. This translates into the magnetic charge carriers forming a dipole layer in the vicinity of the surface $\Sigma$ to prevent the penetration of the external magnetic field into the magnetic conductor, which is the vacuum of QCD(adj) from Euclidean viewpoint. Since $\langle W_{\rm odd}(C)\rangle = e^{-TA(\Sigma)}$ where $T$ is string tension, $T \equiv S^*$,

$$T \sim \Lambda^2(\Lambda L)^{b_0 - 2} = \Lambda^2(\Lambda L)^{(5-2n_f)/3} \; . \tag{16}$$

This exhibits the area law of permanent quark confinement in QCD(adj) in the $L\Lambda \ll 1$ regime. We expect the tension to saturate to a c-number times $\Lambda^2$ for $L\Lambda > 1$.

On the other hand, $0 \equiv 2\pi$ monodromy can be induced by no-soliton, and even soliton sector of the dual theory. Hence, $\langle W_{\rm even}(C)\rangle = 1 + O(e^{-2T{\rm Area}(\Sigma)})$, and no area law as expected. (In the strongly coupled regime, this must become perimeter law.)

**Chiral symmetry realizations:** At small $S^1$, we will argue that the only broken symmetry is the discrete chiral symmetry (which is intertwined with $\mathbb{Z}_2$ shift symmetry of photon and we already showed this), and no continuous chiral symmetry is broken. Consider for simplicity the theory with $n_f = 2$ flavors. Since $\sigma$ is massive, at low energies, the appropriate lagrangian (around $\sigma = 0$ ) is

$$L_{\rm NJL} = i\bar{\psi}^I \gamma_\mu \partial_\mu \psi^I + c e^{-S_0} (\det_{I,J} \psi^I \psi^J + {\rm c.c.}) \qquad (17)$$

NJL type [13]. The action is invariant under the flavor symmetry ($SU(2) \times \mathbb{Z}_4$). We wish to know whether it is spontaneously broken. The $d = 3$ dimensional NJL models has generically two phases depending on the coefficient of the fermion self-interaction $g$ in units of cut-off. In the $g \sim 1$ regime, NJL models exhibit a chiral transition from a chirally symmetric phase at weak coupling to a chirally asymmetric phase in the strong coupling $g > 1$. (See, the review Ref.[14]). Our dimensionless coupling constant is $g \sim e^{-S_0}$, which is a tiny number. Hence, the chiral symmetry must be unbroken, and there must be massless fermions (protected by chiral symmetry) in the spectrum within the region of validity of our long distance effective theory, ($L\Lambda \ll 1$). We believe this is true for $n_f > 2$, as well.

The unbroken continuous chiral symmetry does not exclude the presence of flavor singlet chiral condensates. Such an operator is $\det {\rm tr} \lambda^I \lambda^J$. The calculation is slightly technical, but we can estimate it on physical grounds. It must be proportional to $e^{-S_0} L^{-3n_f}$. The $e^{-S_0}$ reflects the fact that it is due to the one monopole sector. And the deceptive UV divergence in the effective lagrangian Eq.17 is cut-off by the finite size of the monopoles in the full theory. Factoring out $\Lambda^{3n_f}$, the expected behavior on $\mathbb{R}^4$, we obtain

$$\langle \Omega_k | \det {\rm tr}\, \lambda^I \lambda^J | \Omega_k \rangle \sim \Lambda^{3n_f} (\Lambda L)^{\frac{11}{3}(1-n_f)} e^{\frac{i2\pi k}{2}} \qquad (18)$$

and the phase is $\mathbb{Z}_2$ valued. Hearteningly, this produces the correct $L$ independence in the $n_f = 1$ case, which is just $\mathcal{N} = 1$ SYM [15], and two isolated vacua.

At large $S^1$ (and $\mathbb{R}^4$), the common lore is that the chiral symmetry is spontaneously broken down to $SO(n_f) \times \mathbb{Z}_2$, hence there are two isolated coset spaces each of which is $SU(n_f)/SO(n_f)$, and distinguished from each other by the phase of $\langle \det {\rm tr}\, \lambda^I \lambda^J \rangle \in \mathbb{Z}_2$. This implies QCD(adj) must possess a (nonthermal) chiral phase transition in the absence of any change in its spatial center symmetry realization. (The existence of the transition can be proven by considering the theory on $T^2 \times \mathbb{R}^2$, and using Coleman's theorem [16] in small $T^2$ limit.) This is a quantum phase transition at absolute zero temperature, of which there are many examples in condensed matter physics.

*Remark:* We believe the naive extrapolation of the NJL Lagrangian Eq.17 will also exhibit this transition, and moreover, the transition will take place in an expected regime of QCD. However, this will happen outside the region of validity of our effective theory. Consequently, this does not tell us that monopole induced vertex is the dynamical origin of continuous chiral symmetry breaking.

**Conclusions:** The absence of thermal fluctuations and the fact that quantum fluctuation favoring the unbroken center symmetric vacuum in the weakly coupled regime is the key which makes nonperturbative dynamics of QCD(adj) formulated on spatial $S^1 \times \mathbb{R}^3$ analytically tractable. This provides us one of the few examples of four dimensional gauge theory dynamics which can be understood at a quantitative level, and we are optimistic of further progress. A detailed discussion of microscopic derivations and $SU(N)$ generalization is in preparation.

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